Spatial Extreme Value Analysis of Sea-Level Time-Series

Brian White[1] Richard L. Smith[2] Brian Blanton[3] Rick Luettich[4]

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[1]Ph.D. Student|Department of Statistics & Operations Research|UNC

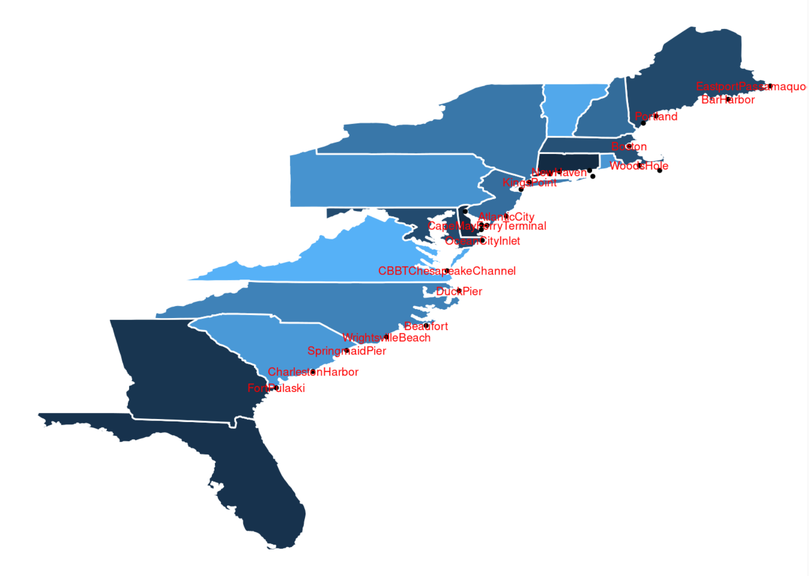
[2]Distinguished Professor|Department of Statistics & Operations Research|UNC

[3]Director of Environmental Initiatives|Renaissance Computing Institute

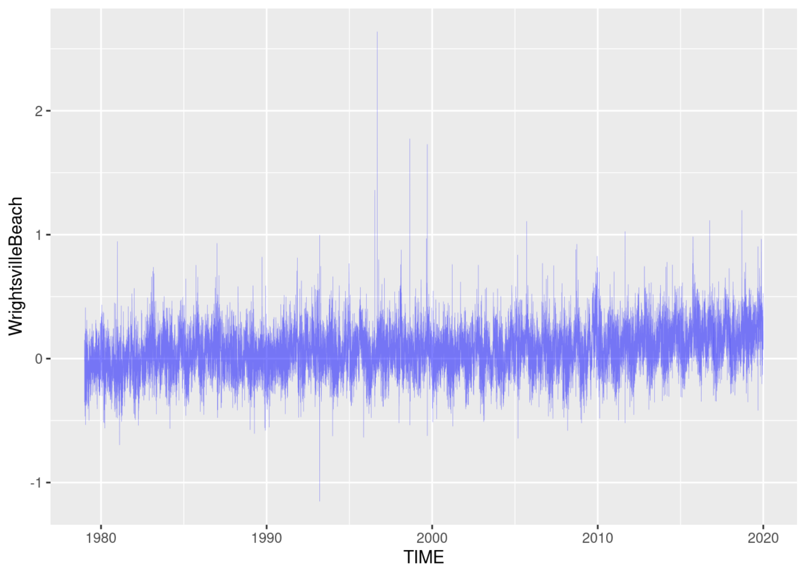
[4]Director|Institute of Marine Sciences|UNC

1. Research aim: Develop a statistical model for the long term spatio-temporal trends in sea-level extremes under different climate-change scenarios.
2. Tools: Spatial statistics, extreme value theory, hierarchical Bayesian modeling, domain expertise.
3. Data:
   (a) Sea-level time-series taken from NOAA observation stations along the U.S. Eastern Seaboard.
   (b) Corresponding model-generated (ADCIRC) sea-level time-series data.

NOAA Stations: FEMA Region 3



EDA



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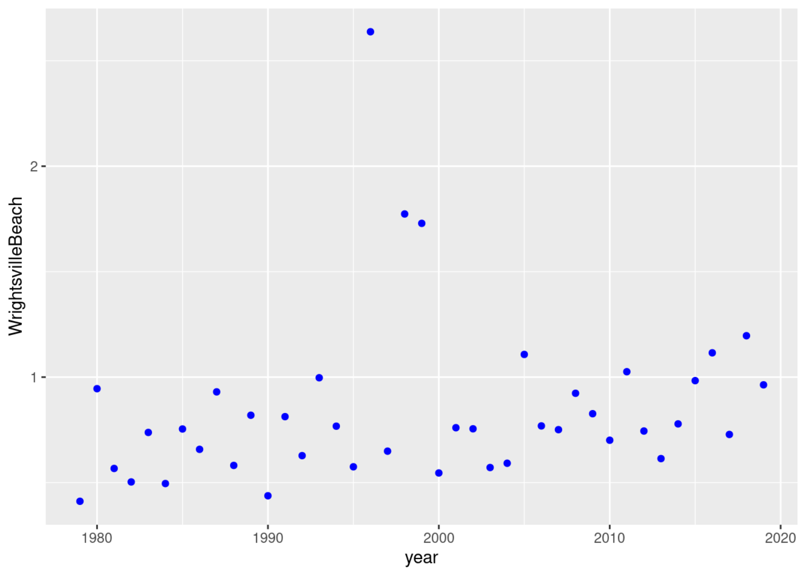

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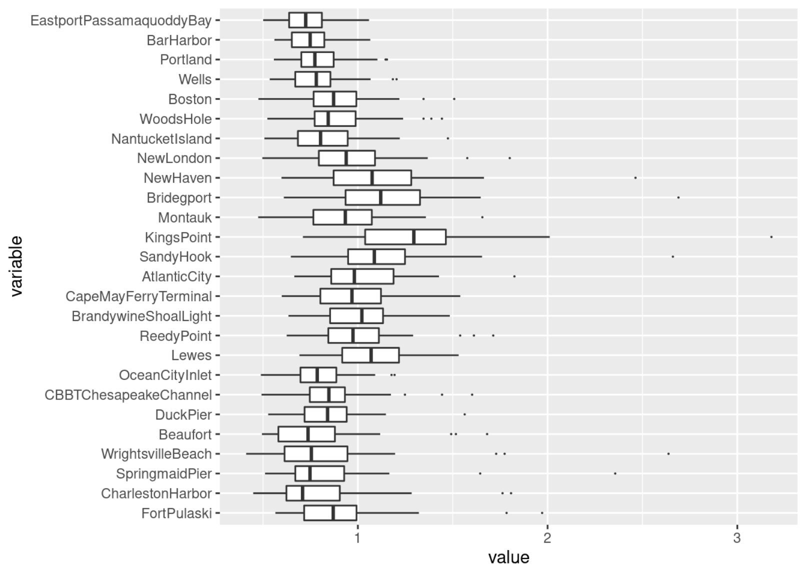
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BarHarbor
Portland
Wells
Boston
WoodsHole
NantucketIsland
NewLondon
NewHaven
Bridgeport
Montauk
KingsPoint
SandyHook
AtlanticCity
CapeMayFerryTerminal
BrandywineShoalLight
ReedyPoint
Lewes
OceanCityInlet
CBBTChesapeakeChannel
DuckPier
Beaufort
WrightsvilleBeach
SpringmaidPier
CharlestonHarbor
FortPulaski
variable
1
2
3
value

Generalized Extreme Value (GEV) Distribution

Let $X_1, X_2, ..., X_n$ be i.i.d. random variables with common CDF F and let $M_n := \max\{X_1, ..., X_n\}$.

Then, when appropriately centered and scaled, M_n converges in distribution to a member of the GEV family:

$$G(z) := \exp\{-[1 + \xi(\frac{z - \mu}{\sigma})]^{-\frac{1}{\xi}}\} \tag{1}$$

- μ is the location parameter
- σ is the scale parameter
- ξ is the shape parameter

GEV Return Levels

$$z_p := \begin{cases} \mu - \frac{\sigma}{\xi}[1 - \{-\log(1-p)\}^{-\xi}] & \xi \neq 0 \\ \mu - \sigma \log\{-\log(1-p)\} & \xi = 0 \end{cases} \quad (2)$$

- $G(z_p) = 1 - p$
- z_p is the return level associated with return period $\frac{1}{p}$
- z_p is expected to be exceeded, on average, once every $\frac{1}{p}$ years.
- In other words, z_p is exceeded by the annual maximum in any given year with probability p.

Modeling Questions

1. How do the GEV parameters depend upon the spatial location?
2. How should one account for dependence among the stations?
3. How should one take into account climate change?

Possible Solution

Idea (Risser 2020): Model the return values spatially with a univariate spatial Gaussian process.

$$z(\mathbf{s}) = y(\mathbf{s}) + \epsilon(\mathbf{s}) \tag{3}$$

- $z(\mathbf{s})$ is the MLE of the return value for location $\mathbf{s}$.
- $y \sim GP(\mu, C_y(.,.;\theta_y))$ is a spatial random effect
- $\epsilon \sim N(0, \tau^2(\mathbf{s}))$ is a stochastic component that represents measurement error or micro-scale variability.

Future Work

1. Implement and extend the model from Risser (2020).
2. Reconcile the ADCIRC-generated and NOAA datasets.
3. Incorporate different climate change forecasts and compare results.

References

1. An Introduction to Statistical Modeling of Extreme Values. Coles. Springer Series in Statistics Springer-Verlag, London, 2001.
2. Investigating the association between late spring Gulf of Mexico sea surface temperatures and US Gulf Coast precipitation extremes with focus on Hurricane Harvey. Russel, Riser, Smith and Kunkel. Environmetrics 31(1), 2019.
3. Non-stationary Bayesian modeling for a large data set of derived surface temperature return values. M. Riser. Lawrence Berkeley National Lab, 2020.